PH102 Tutorial Sheet 2 (Jan 09, 2015)
Department of Physics, IIT Guwahati

Professor Alika Khare & Professor Pratima Agarwal

1. The volume charge density n with in a certain region is given by $n = z^2 sin\varphi$ coulomb/m^3. Calculate the total charge with in the region $0 \le \rho \le 5$, $0 \le \varphi \le \pi$ and $-1 \le z \le 1$.
2. Find the gradient of the following scalar field:
   i. $U = 4xz^2 + 3yz$
   ii. $T = 5\rho e^{-2z} sin\varphi$
   iii. $Q = \frac{sin\theta sin\varphi}{r^2}$
3. The temperature in an auditorium is given by $T = x^2 + y^2 - z$. A mosquito located at a point P (1,1,2) in the auditorium wants to fly in such a direction to get warm as soon as possible. In what direction it should fly?
4. The scalar field in the cylindrical coordinate system is given by $f(\rho, \varphi, z) = \rho cos^2\varphi + z sin\varphi$. Calculate the gradient in spherical polar coordinates.
5. For the function $(x, y, z) = x^2y + yz$, find the rate of change of the function with distance along the direction $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ at a point at a point p(1,2,-1).
6. The temperature profile of a system as a function of T is given in fig 1 below. Mark the arrow head in the direction of gradient of T ($\overrightarrow{\nabla T}$) for the points A, B, C D and E. At what point gradient is maximum?

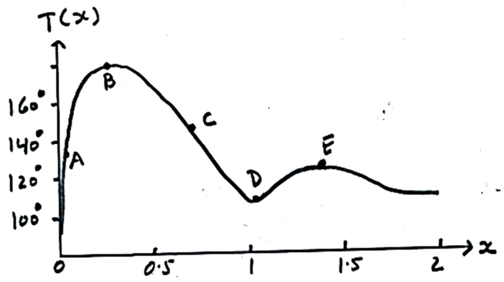

Fig 1

7. Find the divergence of the following vectors:
   i. $\vec{\boldsymbol{A}} = e^{xy}\hat{\boldsymbol{\imath}} + sinxy\hat{\boldsymbol{\jmath}} + cos^2xz\hat{\boldsymbol{k}}$
   ii. $\vec{\boldsymbol{B}} = \rho z^2 cos\varphi\hat{\boldsymbol{\rho}} + z sin^2\varphi\hat{\boldsymbol{k}}$
   iii. $\vec{\boldsymbol{C}} = r cos\theta\hat{\boldsymbol{r}} - \frac{1}{r} sin\theta\hat{\boldsymbol{\theta}} + 2r^2 sin\theta\hat{\boldsymbol{\varphi}}$
8. Find the curl for all the vectors of problem 7 above.
9. Find $\boldsymbol{\nabla}$X($\vec{\boldsymbol{A}}$X$\vec{\boldsymbol{r}}$) where $\vec{\boldsymbol{A}}$ is a constant vector. (Hint: solve the problem in Cartesian coordinates.)
